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On-line identification of operational loads using exogenous inputs

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Abstract

When the FRF matrix describing the dynamical behavior of a structure is available, the operational loads can be determined by multiplying the pseudo-inverse of the FRF matrix by the operational responses (displacements, velocities or accelerations). In practice, however, the boundary conditions of the structure in operation deviate from the ones in laboratory conditions (due to e.g. aerodynamic loads, fuel consumption, temperature changes). This means that measurements during operation should be taken in order to obtain the correct FRF matrix. Unfortunately, it is not always possible to measure all operational loads acting on the structure (which is needed to calculate the FRFs).

In this paper, a method is proposed that enables the on-line determination of operational forces. As input the method uses dynamical response measurements and the measurement of a known force (due to an exogenous excitation input) at one particular location (where it is possible to put an excitation device and a force sensor). A periodic signal is taken as the exogenous excitation. It is assumed that apart from the known force there is also an unknown force (at an unknown location) that is acting on the structure. As a first step in the procedure, the measured responses and the known (i.e. measured) force are compensated in order to eliminate the contribution due to the unknown force. From these compensated measurements the complete FRF matrix is calculated. Then, the forces are calculated from the original (uncompensated) responses and the inverted complete FRF matrix. The method is validated both on a simulation and measurements of a steel beam with an applied unknown impact excitation.

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1. Introduction

During the last two decades many methods were developed to estimate the forces acting on a structure starting from experimentally determined responses of the structure (see the overview in Ref. [1] or more recent references in Ref. [2]). Most of the methods rely on a numerical model (e.g. a FEM model, a spectral element model, etc.) to solve the inverse problem of determining the forces. More recently, a force localization method was proposed that uses only experimental data [3,4]. The method described in Ref. [3] contains two steps:

(1) Firstly, a modal analysis is performed (beforehand in laboratory conditions) in order to determine the complete modal model and the complete re-synthesized FRFs [5].
(2) The experimentally measured responses are multiplied by the weighted pseudo-inverse of the complete FRFs to obtain the force.

Because of the two-step approach, the method only works well when the boundary conditions in Step 1 are the same as in Step 2. Due to the influence of the operational environment this is usually not the case (e.g. presence of aerodynamic loads, fuel consumption, temperature changes). In Ref. [6], a force identification method based on operational modal analysis was proposed in order to solve the problem in one step (i.e. without requiring a laboratory modal test). Although the results in Ref. [6] were very promising, the method only works when the operational loads have a flat broadband spectrum (e.g. not for sines, narrow-band signals, etc.). In addition, in order to be able to scale the mode shapes one has to be able to apply a mass onto the structure.

In this paper, a method is proposed to estimate the location and magnitude of a force from measurements of a force at another location and responses at different locations. The method uses the following three steps:

- Compensate the responses and the measured force to eliminate the contribution of the unknown force.
- Compute the full modal model (and re-synthesize FRFs) from the compensated measurements obtained in the previous step.
- Invert the FRFs and multiply the inverted FRFs with the (uncompensated) responses.

The proposed procedure will be described in detail in Section 2. Validation results on a computer simulation are given in Section 3, and experimental results are given in Section 4. Finally, conclusions are drawn in Section 5.

2. Theory

Assume that $f_k(t)$ is a known (i.e. measured) periodic force at output location i_k (i.e. the result of a periodic exogenous input at a certain location) and that $f_u(t)$ is the unknown operational force at an unknown location i_u (remark that the location of the force f_k is arbitrary but an optimal placement maximizing the signal-to-noise ratio of the measurements could be used [7]). Furthermore, it is assumed that the responses x_i (for $i = 1, \ldots, N_o$) of the system are measured at N_o locations. The dynamical behavior of the system can be characterized by a certain unknown frequency response function matrix $H(\omega)$ as is schematically illustrated in Fig. 1. The goal of the paper is to develop a method which can estimate:

(1) the FRF matrix $H(\omega)$,
(2) the unknown force $f_u(t)$

from the measurements of the known force $f_k(t)$ and the responses $x_i(t)$ for $i = 1, \ldots, N_o$.

To do this the following procedure is used:

(1) Measure two periods of the known periodic force: $f_k(T_s, 2T_s, \ldots, 2T)$ (with $T_s = 1/F_s$ the sample time and $T = NT_s$ the period of the exogenous input signal, which equals the period of the known force).
(2) Apply an FFT on the known force $f_k(T_s, \ldots, 2NT_s)$ and on the responses $x_i(T_s, \ldots, 2NT_s)$ to obtain $X_i(1f_0, 2f_0, \ldots)$ and $F_k(1f_0, 2f_0, \ldots)$ (with $f_0 = 1/(2T)$ the frequency resolution).
(3) Now, because two periods are measured, the even frequency lines of the measured signals $X_i(2f_0, 4f_0, \ldots)$ and $F_k(2f_0, 4f_0, \ldots)$ will have a contribution of both the known and the unknown force while the odd frequency lines $X_i(1f_0, 3f_0, \ldots)$ and $F_k(1f_0, 3f_0, \ldots)$ will only have a contribution of the unknown force.
(4) We assume that there is a correlation between the amplitudes and the phases of the odd and even frequency lines of the unknown force (i.e. it is assumed that the unknown force is not random but e.g. an impulse or a multi-sine). Then, the amplitudes and phases of F_k and X_i at the odd lines $1f_0, 3f_0, \ldots,$ are interpolated in the even frequency lines $2f_0, 4f_0, \ldots.$ The obtained spectral lines are denoted $F_k^{\text{interp}}(2f_0, 4f_0, \ldots)$ and $X_i^{\text{interp}}(2f_0, 4f_0, \ldots)$. In the paper a spline interpolation is used to perform this task(by virtue of the `interp1` function in Matlab with '`spline`' as an argument).
(5) Compensated signals $F_k^{\text{comp}}(2f_0, 4f_0, \ldots)$ and $X_i^{\text{comp}}(2f_0, 4f_0, \ldots)$ are computed by subtracting the energy of the measured signals at the even lines by the interpolated signals in even frequency lines:

$$\begin{aligned} F_k^{\text{comp}}(2nf_0) &= F_k(2nf_0) - F_k^{\text{interp}}(2nf_0), \\ X_i^{\text{comp}}(2nf_0) &= X_i(2nf_0) - X_i^{\text{interp}}(2nf_0). \end{aligned} \tag{1}$$

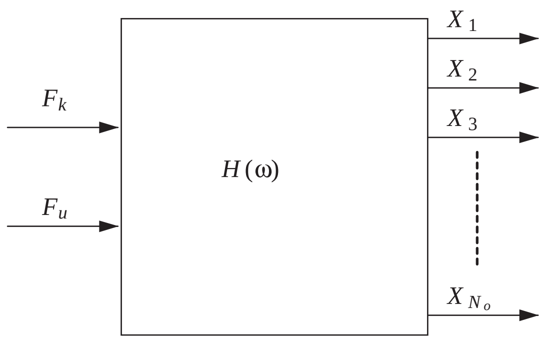

Fig. 1. Input–output model of the system with $H(\omega)$ the FRF, and F_k, F_u and X_i the frequency spectra of forces and responses.

The resulting compensated signals only have a contribution of the known force at the even frequency lines (i.e. the contribution due to the unknown force is eliminated). Further details on the compensation method (for multi-sine signals) can be found in Ref. [8], where compensation was used to eliminate background disturbances.

(6) From the compensated signals $F_k^{\text{comp}}(2f_0, 4f_0, \ldots)$ and $X_i^{\text{comp}}(2f_0, 4f_0, \ldots)$ a parametric model parameter model is estimated. The maximum likelihood estimator presented in Ref. [9] is used for this purpose. Remark that the mode shapes can be scaled because the direct FRF (between the known force and the response) is available when one measures the response at the location of the exogenous input. Using the system poles p_m and scaled mode shapes Φ_m (for $m = 1, \ldots, N_m$ with N_m the number of modes) the FRF matrix is synthesized at all frequency lines $1f_0, 2f_0, 3f_0, \ldots$:

$$\mathbf{H}(f) = \sum_{m=1}^{N_m} \frac{\Phi_m \Phi_m^{\mathrm{t}}}{2\pi \imath f - p_m}. \tag{2}$$

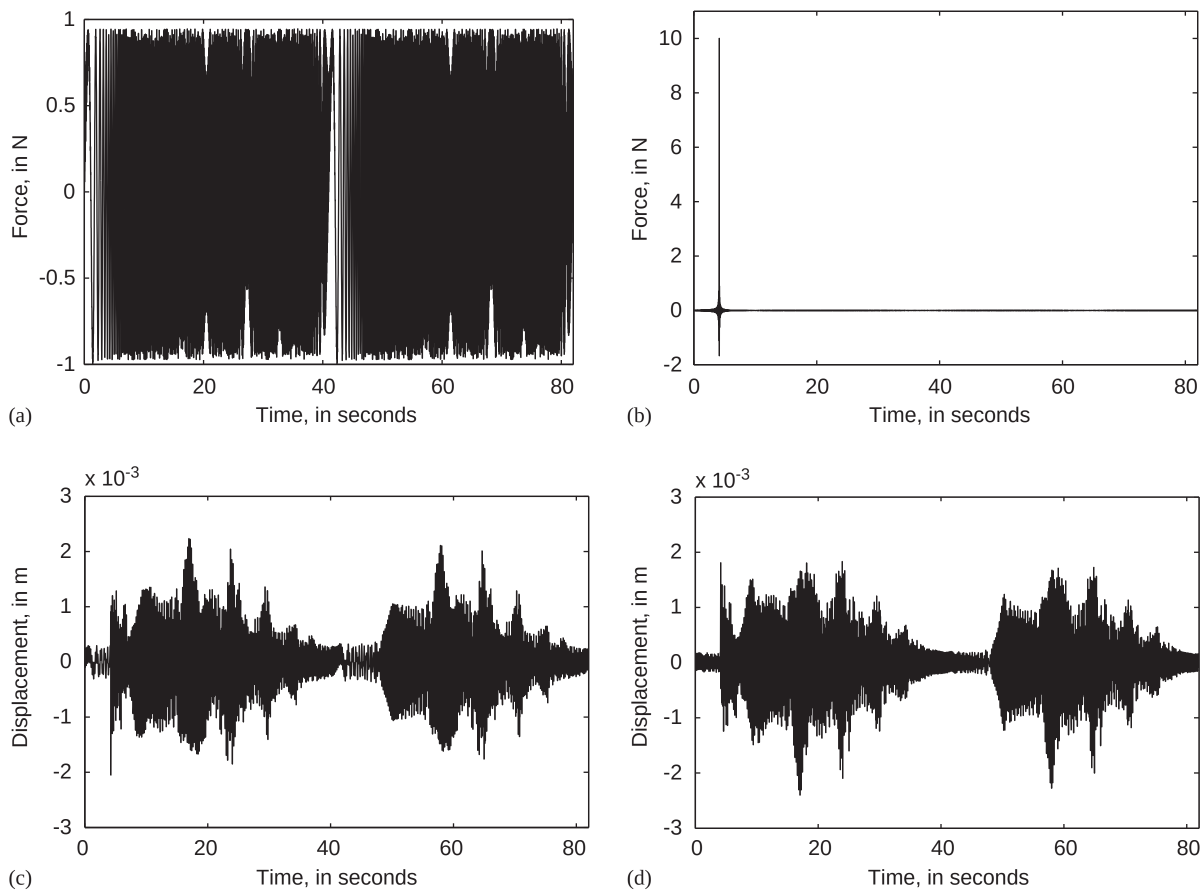

Fig. 2. Time data: (a) known force $f_k(t)$, (b) unknown force $f_u(t)$, (c) displacement at dof 1 $x_1(t)$, (d) displacement at dof 6 $x_6(t)$.

(7) Finally, the weighted pseudo-inverse $\mathbf{H}(f)^+$ of the FRF matrix is calculated (as described in detail in Ref. [3]) and the forces are calculated:

$$\mathbf{F}(f) = \mathbf{H}(f)^+\mathbf{X}(f). \quad (3)$$

In the force vector only two elements i_k and i_u will be non-zero in case of a localized force: $\mathbf{F}_{i_k} = F_k^{\text{est}}$ and $\mathbf{F}_{i_u} = F_u^{\text{est}}$ (the comparison of the estimated F_k^e and the measured F_k is used to validate the method).

It has to be remarked that in order that the problem in Eq. (3) has a localized solution (non-zero forces at both known and unknown force locations) both forces should be applied at one of the output locations. If this is not the case the energy of the identified forces will be smeared out.

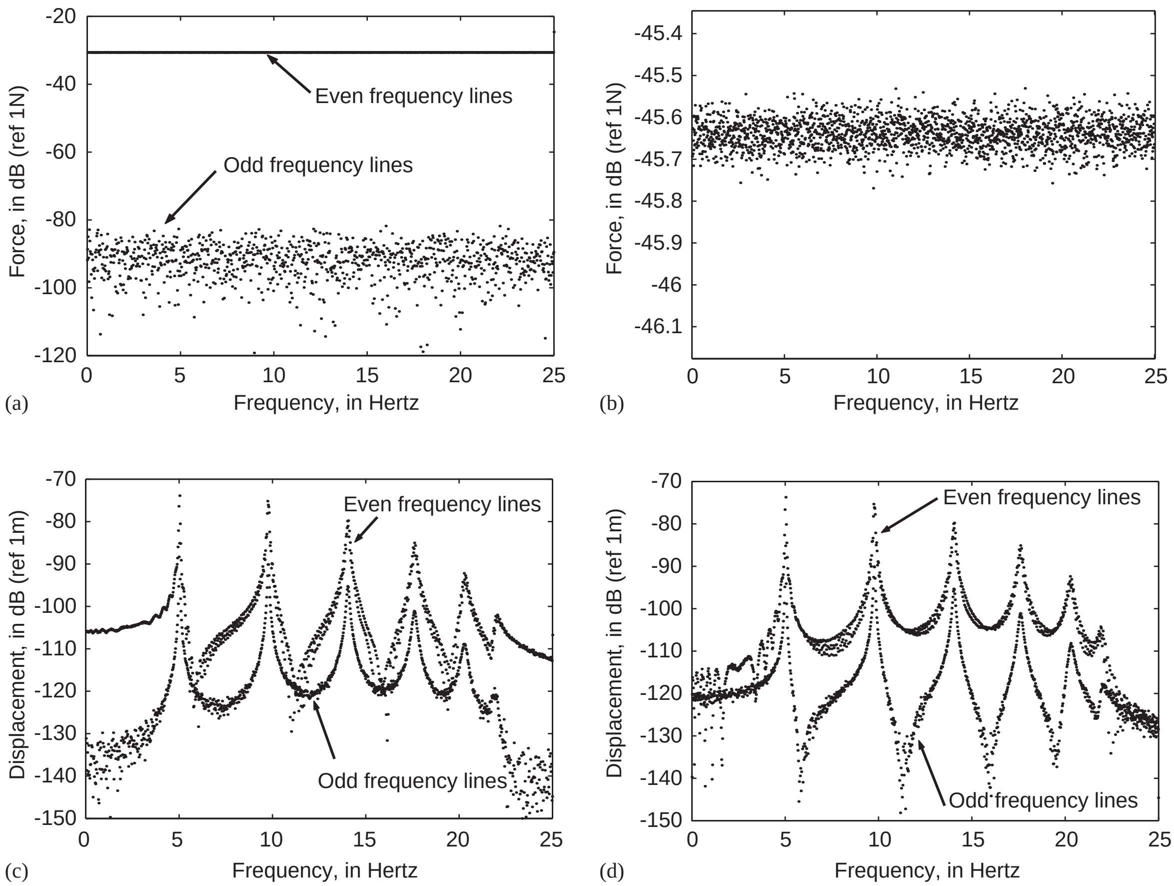

Fig. 3. Frequency domain data: (a) known force $F_k(f)$, (b) unknown force $F_u(f)$, (c) displacement at dof 1 $X_1(f)$, (d) displacement at dof 6 $X_6(f)$.

3. Simulation results

In order to validate the proposed procedure, a six-degree-of-freedom mass–spring chain system is simulated (with $m_i = 1$ and $k_i = 5000$ for $i = 1, \ldots, 6$). The system is excited with an impulse at dof 6 during operation (unknown force $f_u(t)$). Furthermore, a periodic exogenous input force $f_k(t)$ is applied at dof 1 (a multi-sine signal was chosen because of its small crest factor although any periodic signal could be used). Sixty decibel of Gaussian noise is added to the measurements of force and responses.

The time domain signals of the measured forces are given in Fig. 2. Remark that the unknown force $f_u(t)$ in Fig. 2(b) is used only as a reference (it is not used in the computation). In the responses in Figs. 2(c) and (d) it can be seen that around 4 s a contribution due to the impulse in Fig. 2(b) is present. Because of this contribution, the responses are not periodic anymore. This is more clearly seen in the spectra of the time domain signals in Fig. 3. Because force at the

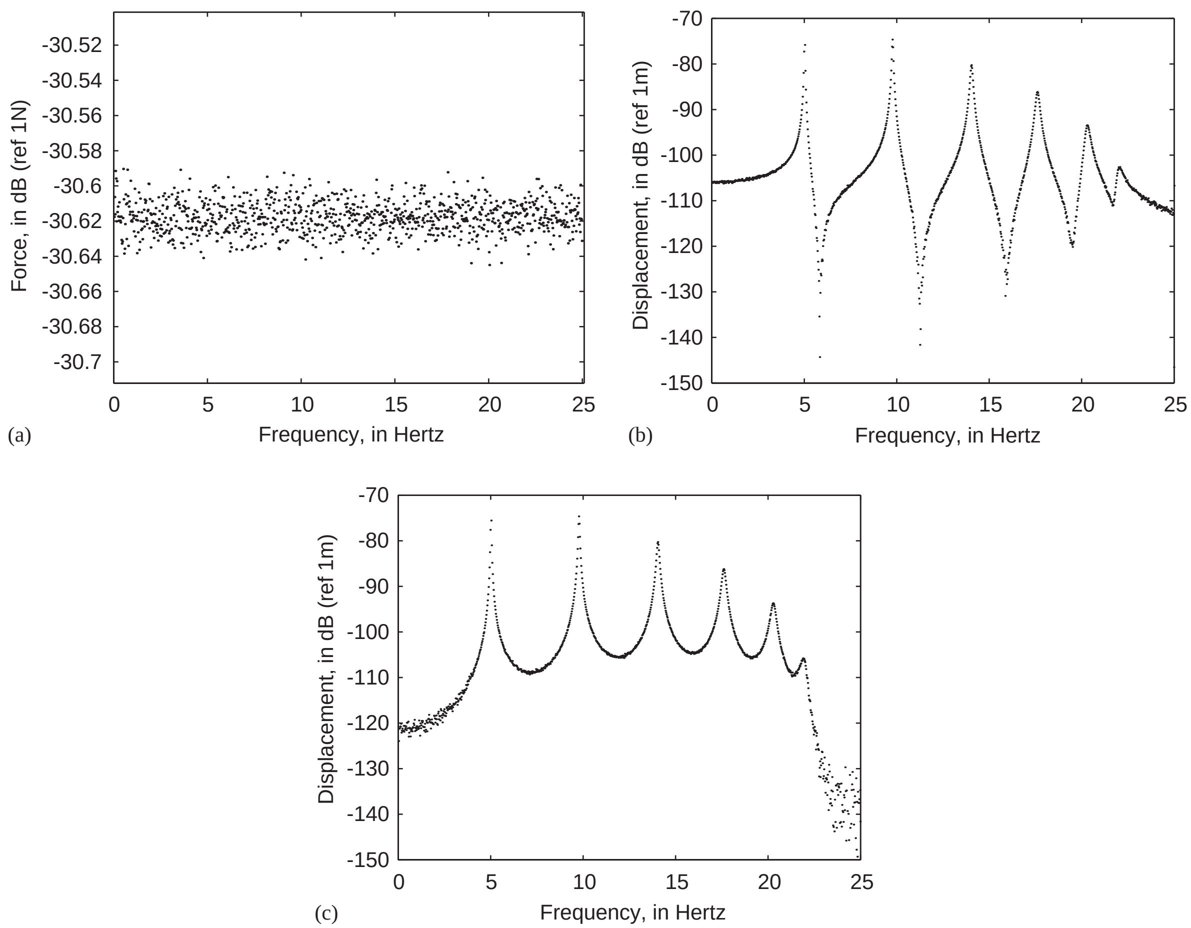

Fig. 4. Compensated frequency domain data: (a) known force $F_k^{\mathrm{comp}}(f)$, (b) displacement at dof 1 $X_1^{\mathrm{comp}}(f)$, (c) displacement at dof 6 $X_6^{\mathrm{comp}}(f)$.

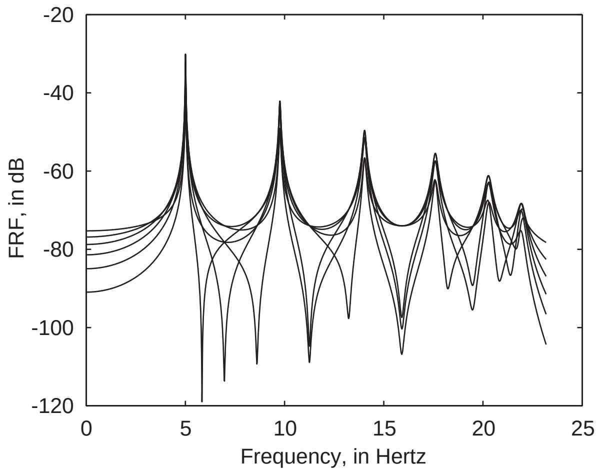

Fig. 5. Re-synthesized FRFs.

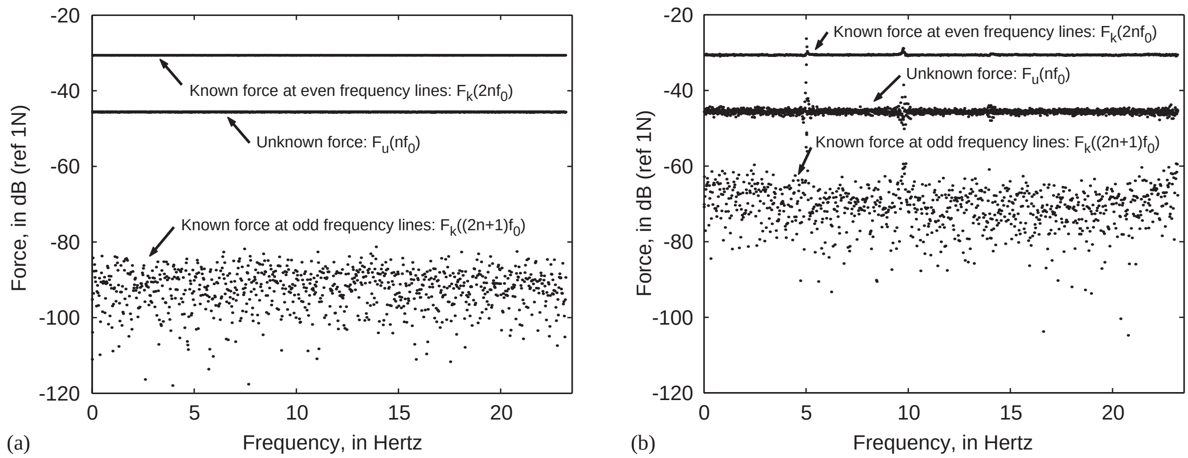

Fig. 6. Frequency domain forces for the computer simulation. (a) True known and unknown forces $F_k(f)$ and $F_u(f)$, (b) estimated known and unknown forces $F_k^{est}(f)$ and $F_u^{est}(f)$.

exogenous input in Fig. 3(a) is purely periodic (two periods are measured) only the even frequency lines are non-zero (the odd lines are at the measurement noise level). The operational force is not periodic and therefore contains energy at both odd and even frequency lines as can be seen in Fig. 3(b).

The responses in Figs. 3(c) and (d) have at the odd frequency lines only a contribution from the operational load and on the even frequency lines a contribution from both forces (exogenous and operational). Using the procedure in Section 2 (Steps 4 and 5 in the algorithm), these contributions can be separated. The resulting compensated signals are shown in Fig. 4. From the compensated signals the FRFs $H_i = X_i/F_k^{comp}$ for $i = 1, \ldots, 6$ can be computed. After computing

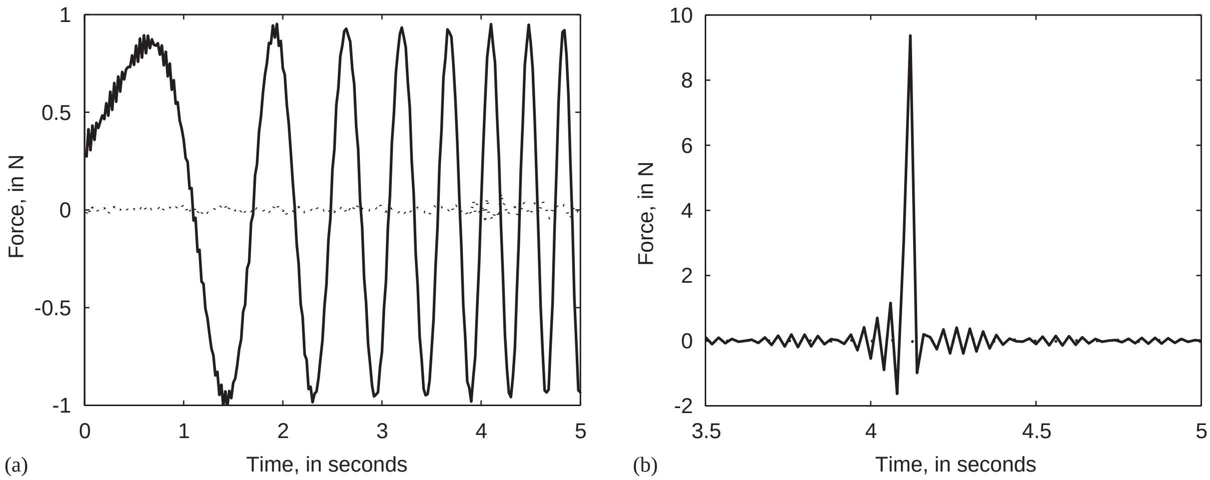

Fig. 7. Time domain forces for the computer simulation. (a) Estimated known force $f_k^{est}(t)$ (full line) and difference between true and estimated known force $f_k^{est}(t) - f_k(t)$ (dotted line) $F_k(f)$, (b) estimated unknown force $f_u^{est}(t)$ (full line) and difference between true and estimated unknown force $f_u^{est}(t) - f_u(t)$ (dotted line).

the modal parameters using the six FRFs a full modal model is obtained and the full FRF matrix can be re-synthesized (six re-synthesized FRFs are shown in Fig. 5).

By calculating the weighted inverse of the FRF matrix and multiplying these inverse FRFs with the responses, the forces are estimated. The results—compared with the true values of the unmeasured forces—are shown in Fig. 6. Globally, there is about 1 dB error on the amplitude of the estimated unknown force. This is quite good keeping in mind that the inverse force identification problem has a bad numerical condition. Near the resonances of the structure up to 10 dB error is obtained (this is due to the interpolation error in the compensation step of the method). When comparing the true and estimated forces in the time domain (see Fig. 7) it can be seen that only a few percent error is made.

4. Experimental results

The experimental validation is performed on a beam which is freely supported. As the operational load, an impact was generated with a calibrated hammer. The exogenous input (a multi-sine with random phases and uniform amplitudes) was applied with a B&K mini shaker and the acceleration responses are measured at six locations with PCB accelerometers (the setup is shown in Fig. 8).

Time domain and frequency domain measurements are shown in Figs. 9 and 10, respectively. Again, the separation between the odd contribution (operational load only) and the even frequency lines (combination of exogenous and operational contribution) is clear.

The compensated responses in Fig. 11 are much clearer (no distortions due to the unknown operational pulsed force). These responses together with the compensated exogenous load are used to compute the complete modal model and the complete FRFs (see Fig. 12).

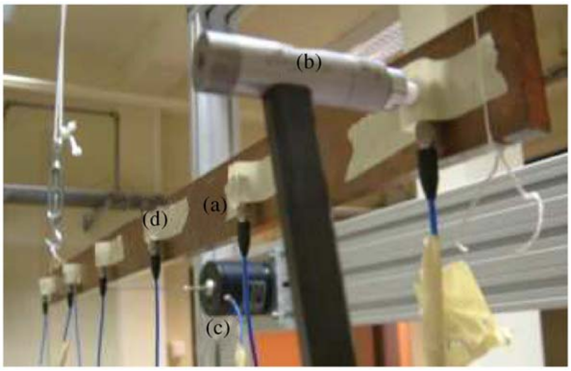

Fig. 8. Measurement setup of the impulse identification experiment: (a) beam under test, (b) impact hammer for application of unknown force f_u, (c) shaker for application of known force $f_k(t)$, (d) acceleration sensors at six equidistant positions.

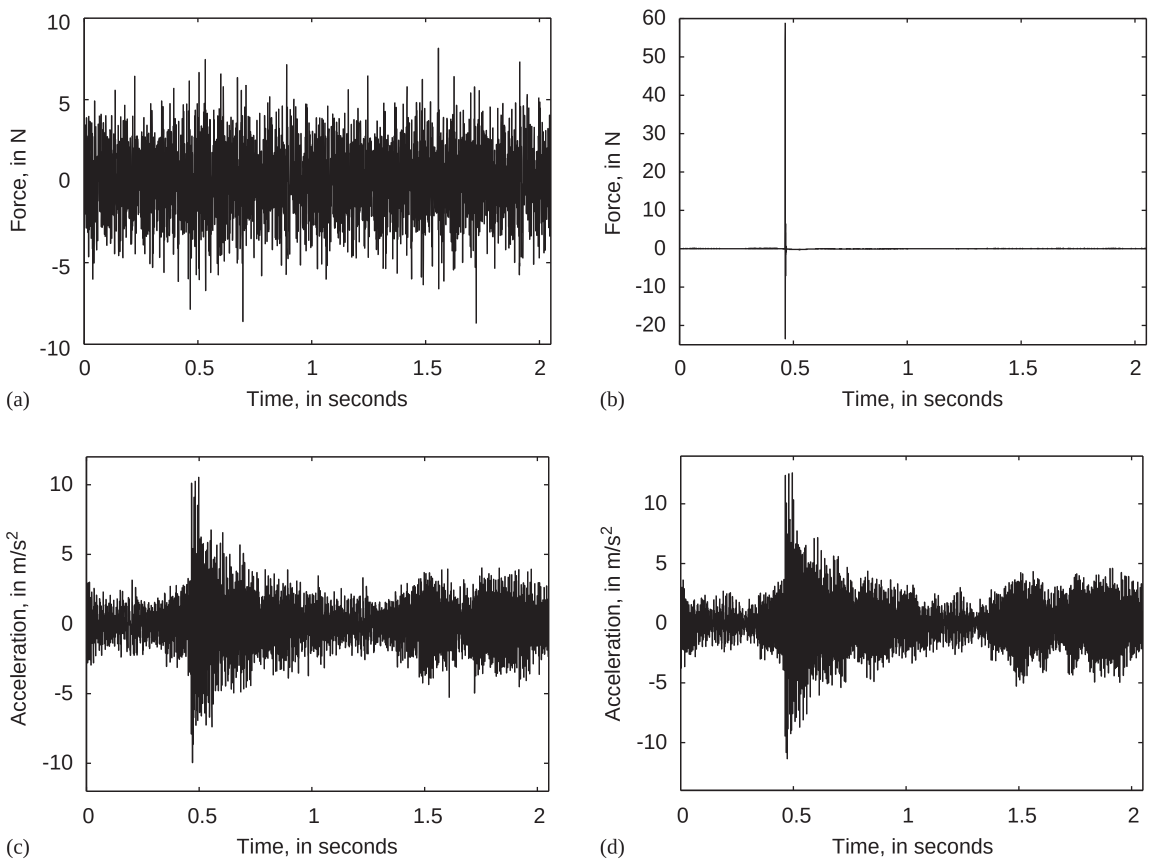

Fig. 9. Time data: (a) known force $f_k(t)$, (b) unknown force $f_u(t)$, (c) displacement at dof 1 $x_1(t)$, (d) displacement at dof 6 $x_6(t)$.

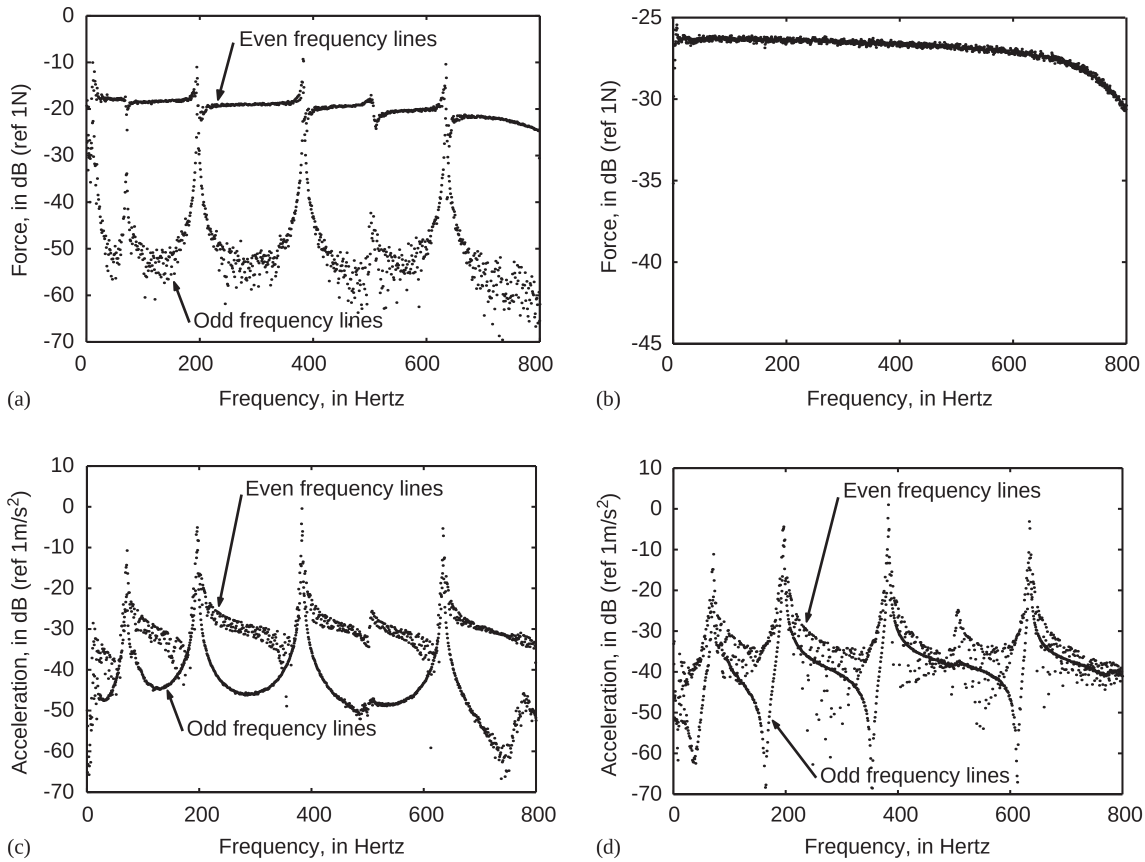

Fig. 10. Frequency domain data: (a) known force $F_k(f)$, (b) unknown force $F_u(f)$, (c) displacement at dof 1 $X_1(f)$, (d) displacement at dof 6 $X_6(f)$.

In the comparison of the spectra of the estimated forces (both known and unknown) with the measured forces in Fig. 13, it can be seen that there is up to 10 dB of distortion on the estimated load amplitudes. In the time domain, the estimated forces (both known f_k and unknown $f_u(t)$) has an error of about 10% compared to the measured force (see Fig. 14). This is much better than one would obtain without compensating for the unknown force in the calculation of the FRFs.

5. Conclusions

In this article a method was developed to estimated operational forces by using an exogenous periodic excitation signal. After compensating the response signals for the presence of the operational load the complete FRF matrix is computed. The inverse of this FRF matrix is then used to calculate the loads. In addition to the unknown operation load, also the exogenous load is applied. This can then be used as a validation by comparing the measured and estimated exogenous forces. Since in interpolation between the phases and

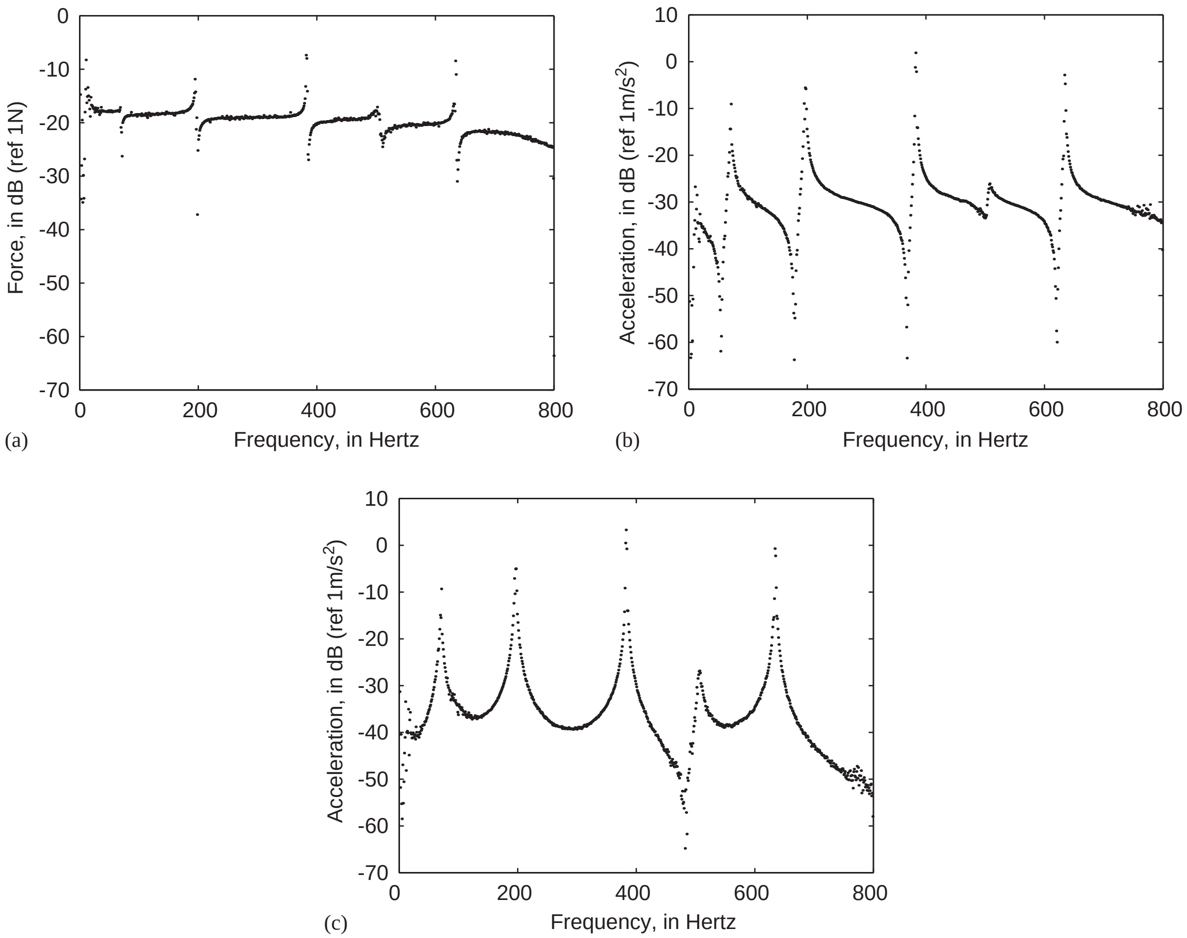

Fig. 11. Compensated frequency domain data: (a) known force $F_k^{\text{comp}}(f)$, (b) displacement at dof 1 $X_1^{\text{comp}}(f)$, (c) displacement at dof 6 $X_6^{\text{comp}}(f)$.

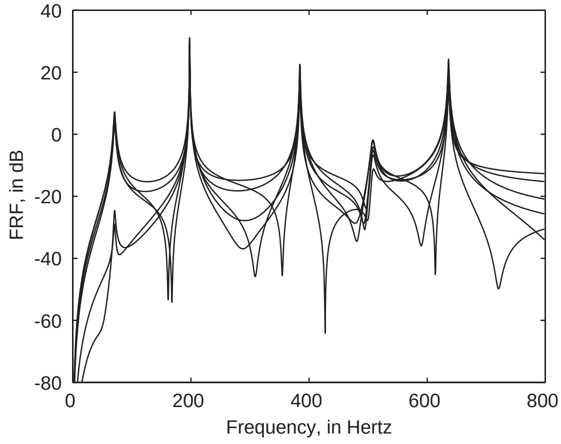

Fig. 12. Re-synthesized FRFs.

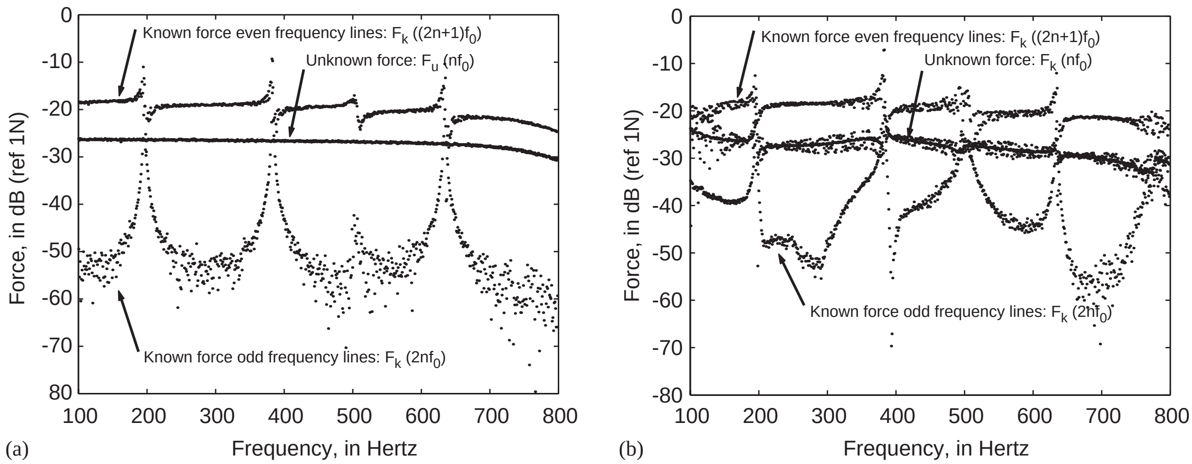

Fig. 13. Frequency domain forces for the beam experiment. (a) True known and unknown forces $F_k(f)$ and $F_u(f)$, (b) estimated known and unknown forces $F_k^{\mathrm{est}}(f)$ and $F_u^{\mathrm{est}}(f)$.

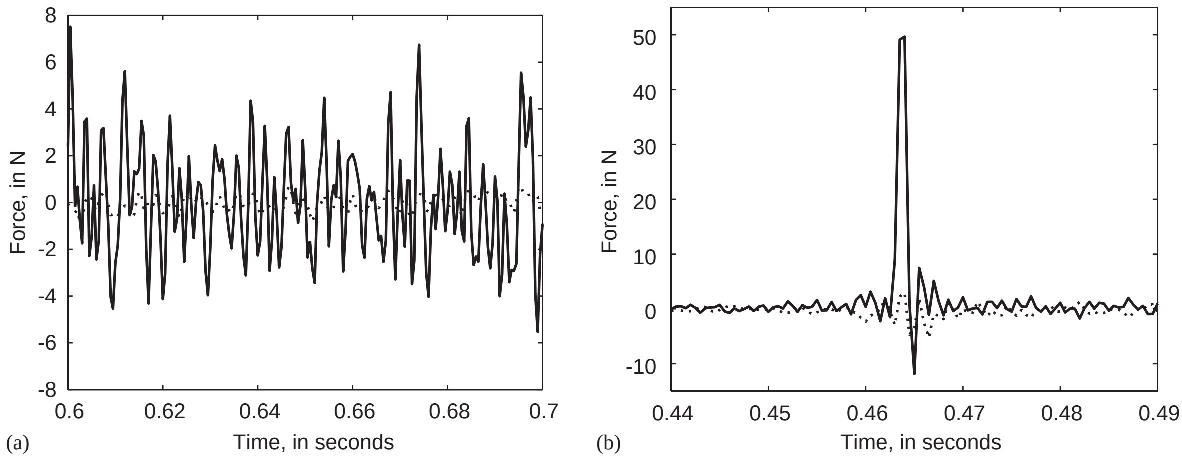

Fig. 14. Time domain forces for the beam experiment. (a) Estimated known force $f_k^{\mathrm{est}}(t)$ (full line) and difference between true and estimated known force $f_k^{\mathrm{est}}(t) - f_k(t)$ (dotted line) $F_k(f)$, (b) estimated unknown force $f_u^{\mathrm{est}}(t)$ (full line) and difference between true and estimated unknown force $f_u^{\mathrm{est}}(t) - f_u(t)$ (dotted line).

the amplitudes of neighboring spectral lines is performed, the method only works for deterministic loads. The accuracy of the estimated forces was 1% (simulation example)–10% (measurement example).

Finally, it can be remarked that, when using a inverted modal model to obtain forces, the model should be very accurate (due to the ill-conditioning of the problem, model errors accumulate). For this reason an accurate maximum likelihood estimator (see Ref. [9]) was used in the paper (other methods like the least-squares complex exponential (LSCE) will work on the laboratory example but errors would be unacceptable on real-life cases).

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